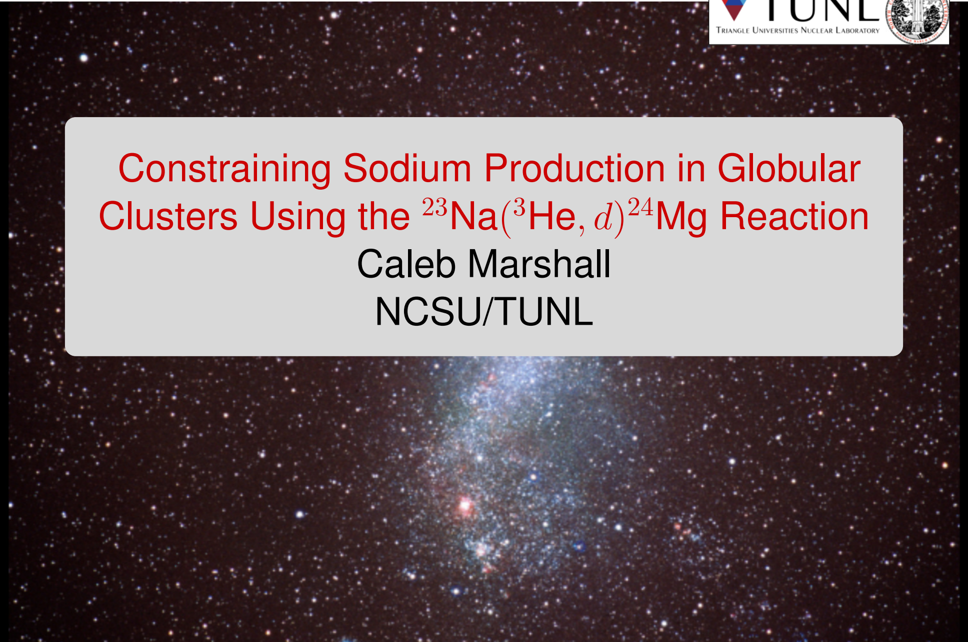


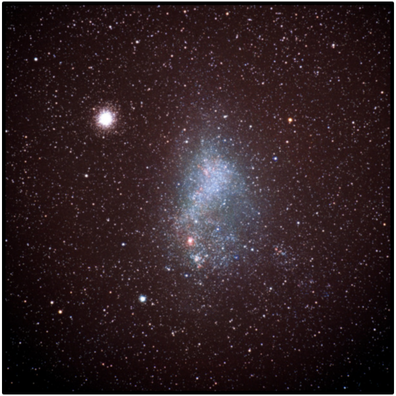
Constraining Sodium Production in Globular Clusters Using the ^{23}Na(^{3}He, d)^{24}Mg Reaction

Caleb Marshall
NCSU/TUNL

Outline

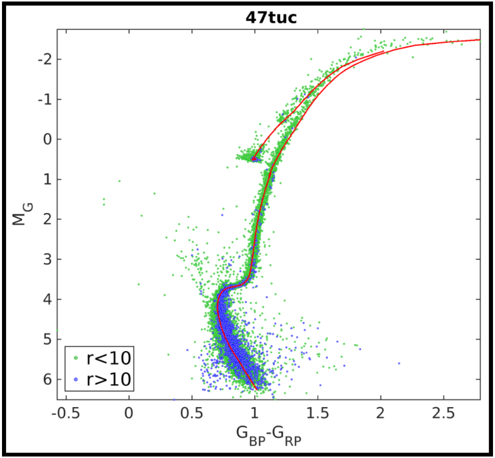
What is a Globular Cluster?

- 10^5 stars gravitationally bound in a small radius.
- Some of the oldest and brightest objects in the galaxy.
- Ideal testing ground for theories of stellar, galactic, and chemical evolution.



47 Tuc next to the small magellanic cloud
https://sci.esa.int/s/AjG4jmw Copyright: Akira Fujii

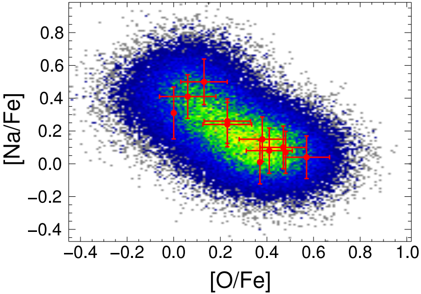
Single Stellar Population



- Only need to specify initial chemical composition and mass distribution.

[1] Gaia Collaboration, Babusiaux, C., van Leeuwen, F., et al. 2018a, A&A, 616,A10

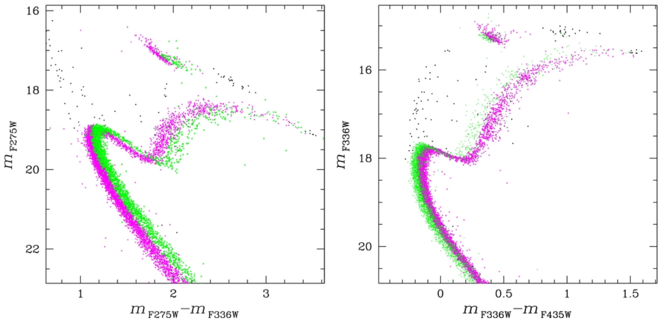
Abundance Anomalies



A.O.Thygesen et al., Astron.Astrophys.572, A108 (2014)

- First discovered in 80's.
- Na-O anticorrelation appears to be ubiquitous.
- Other light element correlations and anticorrelations have been observed.
- Are these a result of initial inhomogeneity in the cluster material?

Multiple Stellar Populations

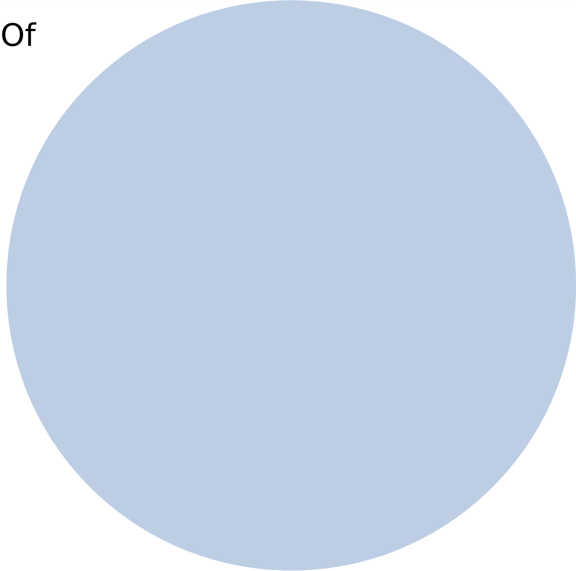


Milone A. P., et al., 2012a, ApJ, 744, 58

- ~ 2007 high resolution photometry reveals multiple main sequences.

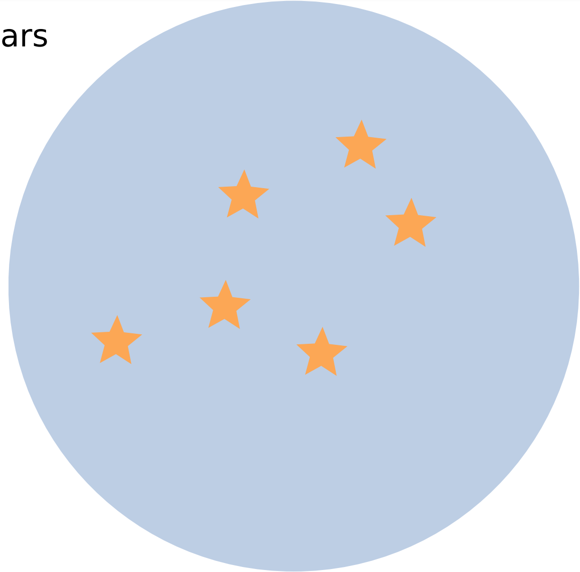
Multiple Populations

Cloud Of
ICM



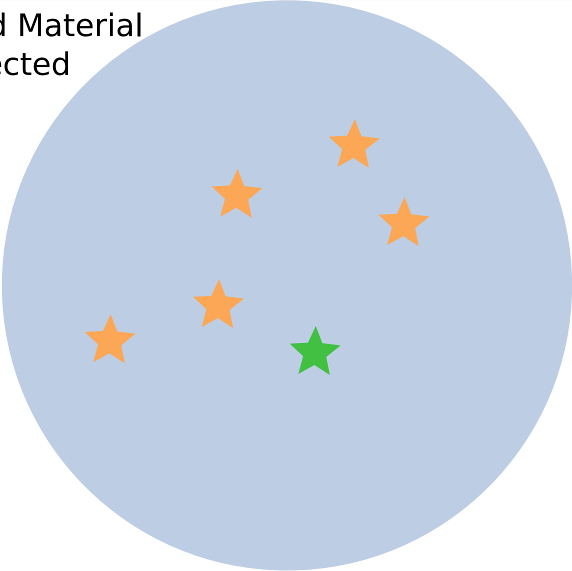
Multiple Populations

First Stars



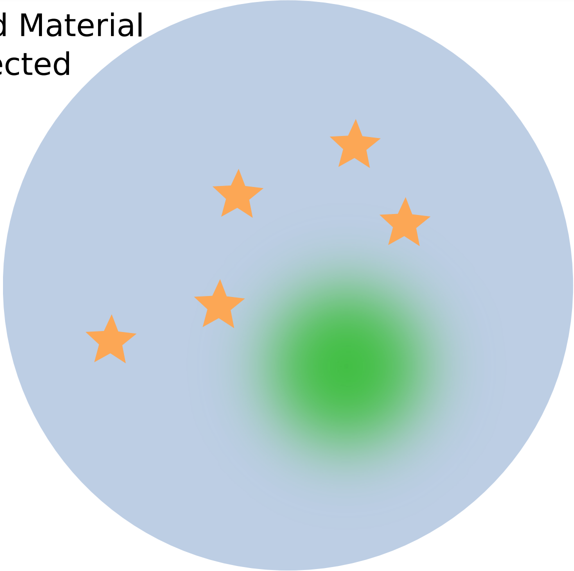
Multiple Populations

Polluted Material
Ejected



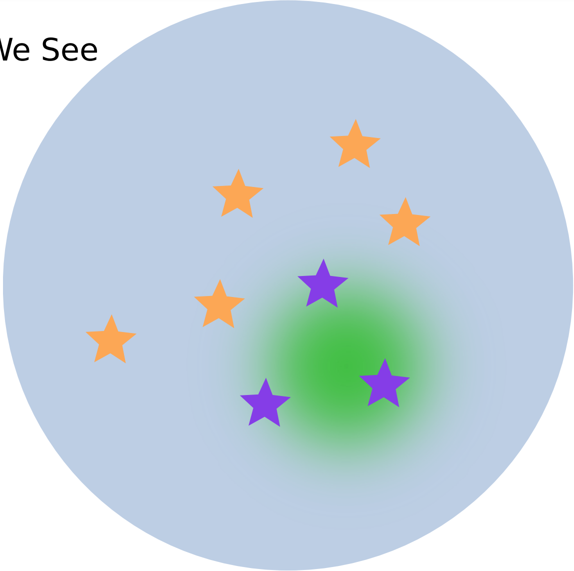
Multiple Populations

Polluted Material
Ejected



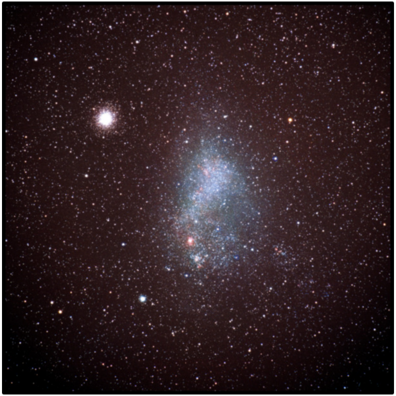
Multiple Populations

Stars We See



What is a Globular Cluster?

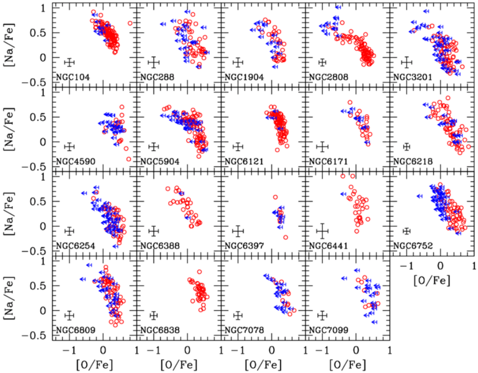
- 10^5 stars gravitationally bound in a small radius.



[1] Gratton, R., et al. 2019, A&A Rv, 27, 8

What is a Globular Cluster?

- Light element variations are the defining feature [1].
- Enriched material comes from older generation of stars.
- Nucleosynthesis is happening *in situ*.

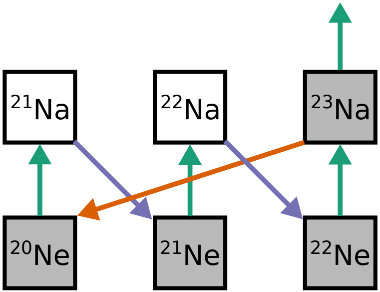


Carretta, E., et al. 2010, A&A, 516

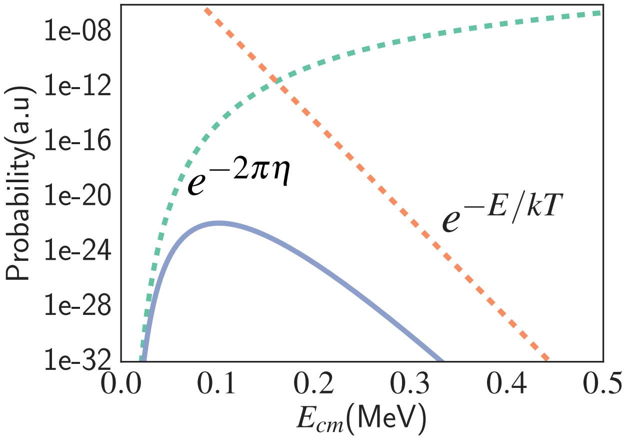
[1] Gratton, R., et al. 2019, A&A Rv, 27, 8

How is a Globular Cluster?

- Where is this enriched material coming from?
- H-burning at 70-80 MK
- ^{23}Na$(p,\gamma)^{24}$Mg only path to heavier elements.
- No environment satisfies all constrains.

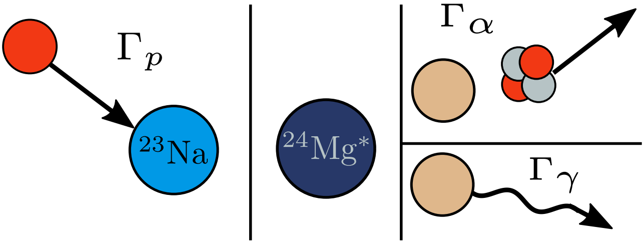


Gamow Window



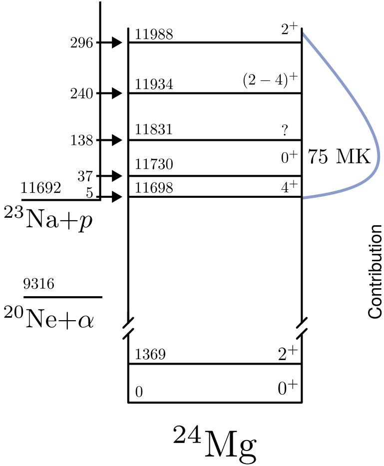
Narrow Resonances

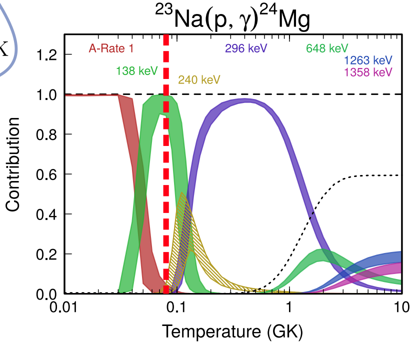
$$\sigma(E)_R \sim \frac{\Gamma_p \Gamma_\gamma}{(E-E_r)^2 + \frac{(\Gamma_p+\Gamma_\gamma+\Gamma_\alpha)^2}{4}}$$



States of Interest

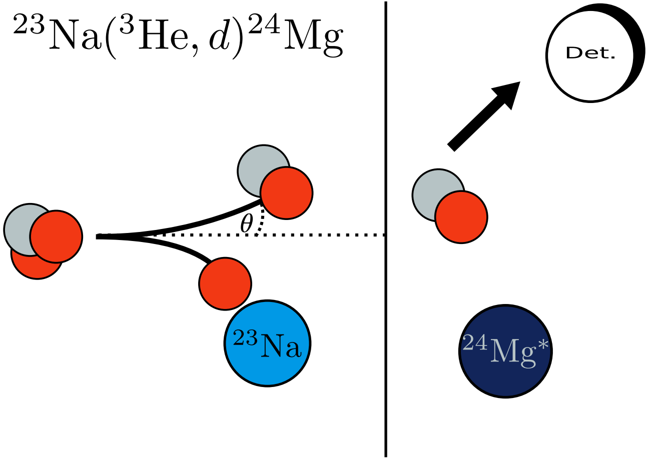
$$N_A < \sigma\nu > \approx \frac{(2J_R+1)}{(2j_t+1)(2j_p+1)} \frac{\Gamma_p \Gamma_\gamma}{\Gamma} e^{-E_r/k_BT}$$





Transfer Reaction

$$^{23}\mathrm{Na}(^{3}\mathrm{He}, d)^{24}\mathrm{Mg}$$



Narrow Resonances

$$N_A < \sigma\nu > \approx \frac{(2J_R+1)}{(2j_t+1)(2j_p+1)} \frac{\Gamma_p \Gamma_\gamma}{\Gamma} e^{-E_r/k_B T}$$

Narrow Resonances

$$N_A < \sigma \nu > \approx \frac{(2J_R+1)}{(2j_t+1)(2j_p+1)} \frac{\Gamma_p \Gamma_\gamma}{\Gamma} e^{-E_r/k_B T}$$

- Shape of angular distribution. $\rightarrow$ Many angles, small $\Delta\theta$.

Narrow Resonances

$$N_A < \sigma\nu > \approx \frac{(2J_R+1)}{(2j_t+1)(2j_p+1)} \frac{\Gamma_p \Gamma_\gamma}{\Gamma} e^{-E_r/k_B T}$$

- Shape of angular distribution. $\rightarrow$ Many angles, small $\Delta\theta$.
- Magnitude of cross section. $\rightarrow$ Absolute scale for data.

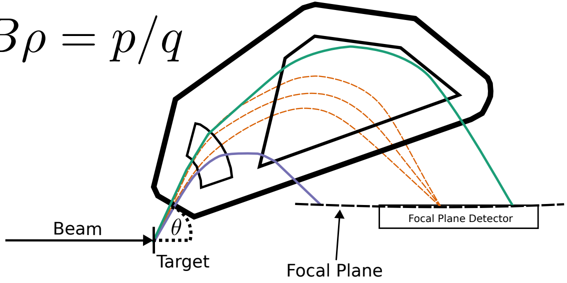
Narrow Resonances

$$N_A < \sigma\nu > \approx \frac{(2J_R+1)}{(2j_t+1)(2j_p+1)} \frac{\Gamma_p \Gamma_\gamma}{\Gamma} e^{-E_r/k_B T}$$

- Shape of angular distribution. $\rightarrow$ Many angles, small $\Delta\theta$.
- Magnitude of cross section. $\rightarrow$ Absolute scale for data.
- Location of the peak $\rightarrow$ High resolution.

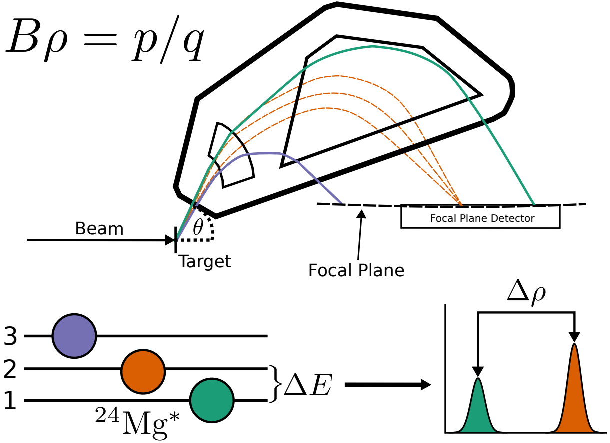
Magnetic Spectroscopy

$$B\rho = p/q$$

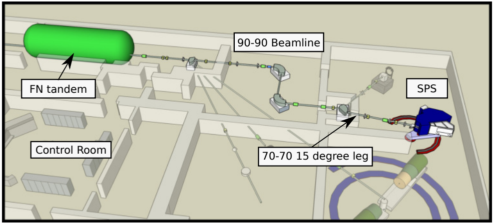


Magnetic Spectroscopy

$$B\rho = p/q$$

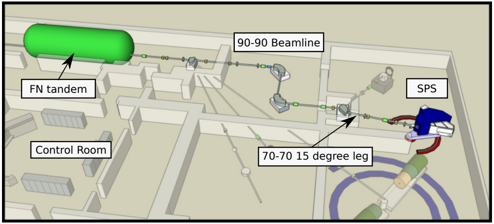


TUNL



- 10 MV FN tandem.

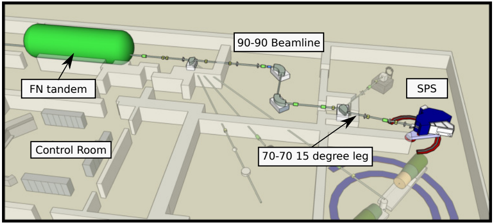
TUNL



- p, d, ^{3}He, and ^{4}He beams readily available.

- 10 MV FN tandem.

TUNL

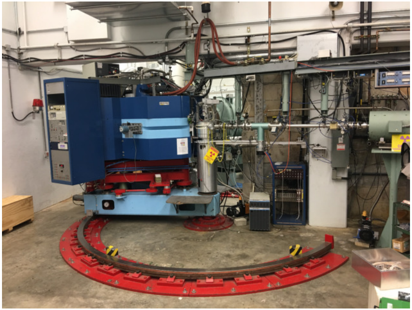


- p, d, ^{3}He, and ^{4}He beams readily available.
- High resolution 90-90 beamline.

- 10 MV FN tandem.

TUNL

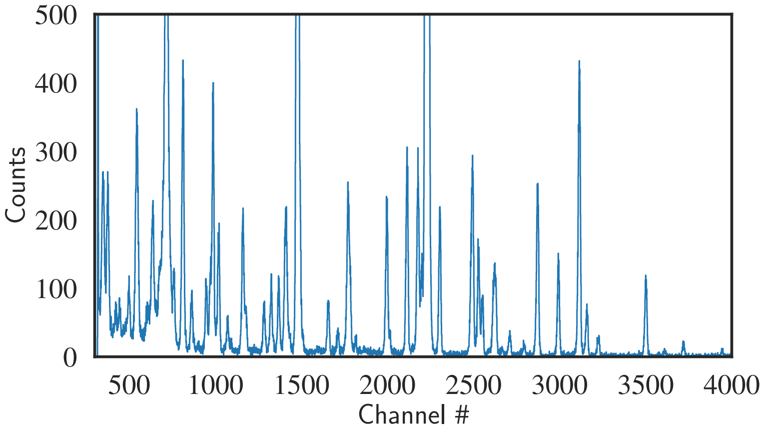
TRIANGLE UNIVERSITIES NUCLEAR LABORATORY



- 10 MV FN tandem.

- p, d, ^{3}He, and ^{4}He beams readily available.
- High resolution 90-90 beamline.
- Angles: 3-$21°$ in $\Delta\theta = 2°$ steps. $E_{Lab} = 21$ MeV.
- A set of 3 NaBr targets were used.

^{23}Na(^{3}He, d)^{24}Mg at 11°



Is ^{24}Mg Really Well Known?

TRIANGLE UNIVERSITIES NUCLEAR LABORATORY

- ENSDF 14 years out of date.

$^{24}_{12}$Mg$_{12}$-1 From ENSDF - Evaluated October 2006 $^{24}_{12}$Mg$_{12}$-1

Adopted Levels, Gammas

Type	Author	History Citation
Full Evaluation	R. B. Firestone	NDS 108, 2319 (2007)

Literature Cutoff Date

3-Oct-2006

$Q(\beta^-)=-13886.0$ *11*; $S(n)=16531.6$ *7*; $S(p)=11692.69$ *1*; $Q(\alpha)=-9316.55$ *1* 2012Wa38
Note: Current evaluation has used the following Q record.
$Q(\beta^-)=-13877$ *3*; $S(n)=16531.1$ *13*; $S(p)=11692.68$ *1*; $Q(\alpha)=-9316.55$ *1* 2003Au03

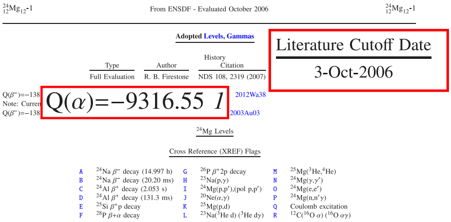
^{24}Mg Levels

Cross Reference (XREF) Flags

A	^{24}Na β^- decay (14.997 h)	G	^{26}P β^+2p decay	M	^{25}Mg(^{3}He,^{4}He)
B	^{24}Na β^- decay (20.20 ms)	H	^{23}Na(p,γ)	N	^{24}Mg(γ,γ')
C	^{24}Al β^+ decay (2.053 s)	I	^{24}Mg(p,p'),(pol p,p')	O	^{24}Mg(e,e')
D	^{24}Al β^+ decay (131.3 ms)	J	^{20}Ne(α,γ)	P	^{24}Mg(n,n'γ)
E	^{25}Si β^+p decay	K	^{25}Mg(p,d)	Q	Coulomb excitation
F	^{28}P β+α decay	L	^{23}Na(^{3}He d) (^{3}He dγ)	R	^{12}C(^{16}O α) (^{16}O $\alpha\gamma$)

Is ^{24}Mg Really Well Known?

- ENSDF 14 years out of date.
- Many levels come from $^{20}\mathrm{Ne}(\alpha,\gamma)$, needed to be updated for Q value.



$^{24}_{12}\mathrm{Mg}_{12}$-1 From ENSDF - Evaluated October 2006 $^{24}_{12}\mathrm{Mg}_{12}$-1

Adopted Levels, Gammas

Type	Author	History Citation
Full Evaluation	R. B. Firestone	NDS 108, 2319 (2007)

Literature Cutoff Date

3-Oct-2006

$Q(\beta^-)$=−138 2012Wa38
Note: Curren
$Q(\beta^-)$=−138 2003Au03

$Q(\alpha)$=−9316.55 *1*

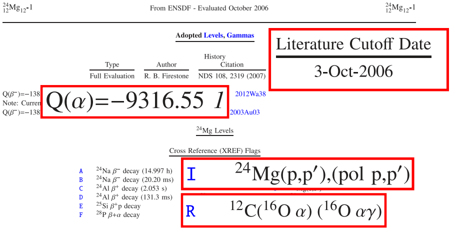
^{24}Mg Levels

Cross Reference (XREF) Flags

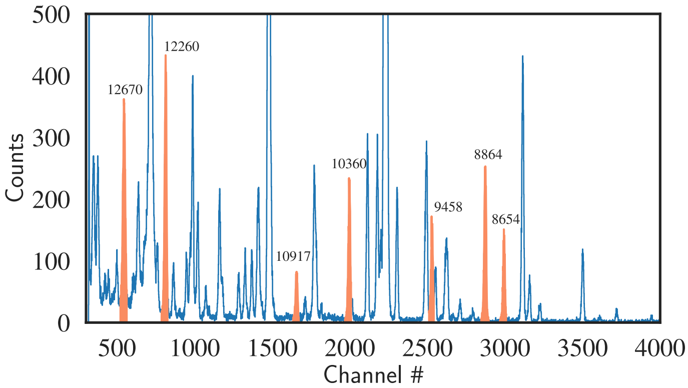
A	^{24}Na β^- decay (14.997 h)	G	^{26}P β^+2p decay	M	^{25}Mg(^{3}He,^{4}He)
B	^{24}Na β^- decay (20.20 ms)	H	^{23}Na(p,γ)	N	^{24}Mg(γ,γ')
C	^{24}Al β^+ decay (2.053 s)	I	^{24}Mg(p,p′),(pol p,p′)	O	^{24}Mg(e,e′)
D	^{24}Al β^+ decay (131.3 ms)	J	^{20}Ne(α,γ)	P	^{24}Mg(n,n′γ)
E	^{25}Si β^+p decay	K	^{25}Mg(p,d)	Q	Coulomb excitation
F	^{28}P β+α decay	L	^{23}Na(^{3}He d) (^{3}He dγ)	R	^{12}C(^{16}O α) (^{16}O $\alpha\gamma$)

Is ^{24}Mg Really Well Known?

- ENSDF 14 years out of date.
- Many levels come from ^{20}Ne(α, γ), needed to be updated for Q value.
- Previous spectrograph measurements use large amounts of calibration states.
- Values are too precise due to deduced gamma ray energies being fed back into least squares fit.

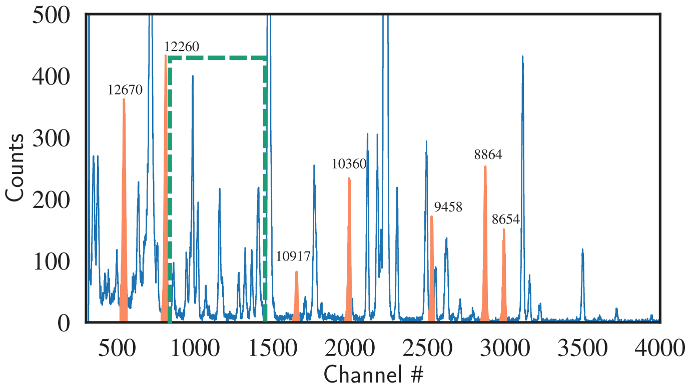


Energy Calibration



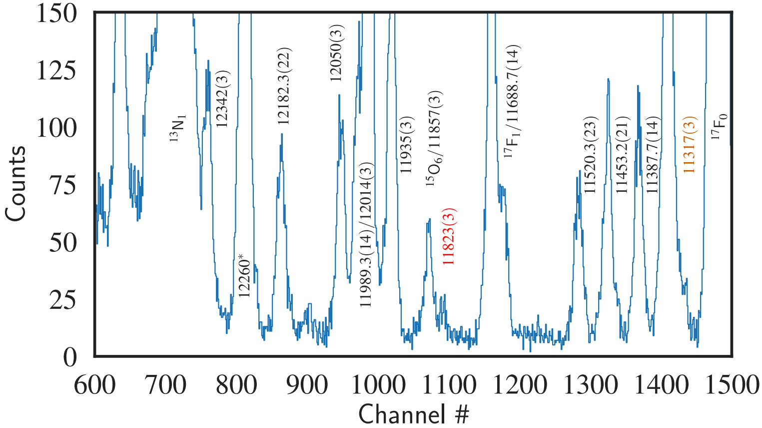
- $\rho = Ax^3 + Bx^2 + Cx + D$
- States selected primarily to avoid closely spaced doublets, $\Delta E < 10$ keV.

Energy Calibration



- States of interest between $\sim 11000 - 12000$ keV.

Energies

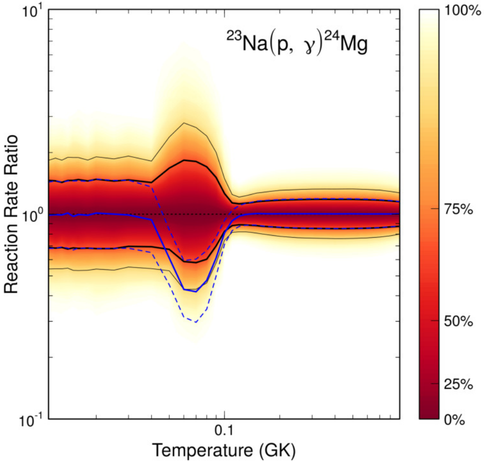


- Our Value: 11823(3) conflicts with previous value of $11831.7(18)$.[1]
- Previous energy depends on identity of 11317.

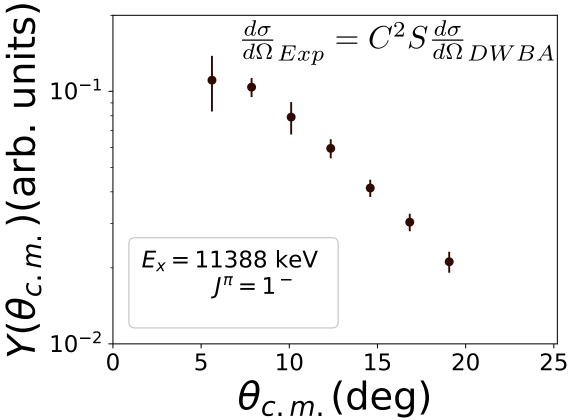
[1] S. E. Hale et al., PRC 70, 2004

Astrophysical Importance

- Recommended energy of $11825(3)$ excludes Hale et al.
- $e^{-E_r/kT}$, means large impact on rate.
- Gamma ray measurements needed to verify these findings.
- Factor of 5 increase in rate.

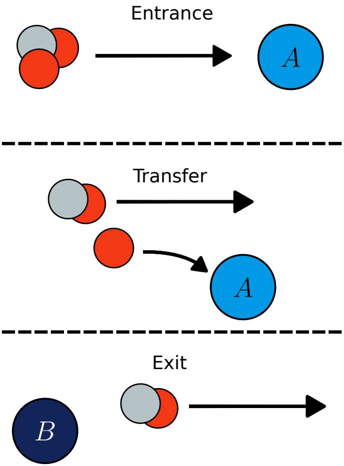


The Need for a Theory



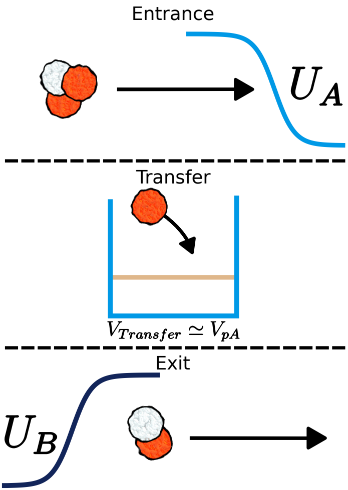
- $\Gamma_p = C^2 S_\ell \Gamma_{sp,\ell}$
- Information about ℓ and C^2S in angular distribution.
- Theory needs to correctly predict the shape and magnitude.

DWBA



- Assuming a direct reaction process for ^{3}He$+A \rightarrow d + B$.

DWBA



- Assuming a direct reaction process for ^{3}He$+A \rightarrow d + B$.
- Distorted-wave Born approximation.
- Distorted-waves describe the elastic scattering channels.
- Particle occupies a single particle state.

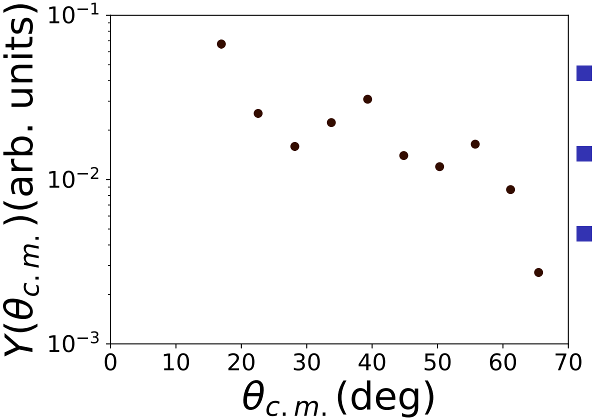
Optical Potentials

$$\mathcal{U}(r) = V_c(r; r_c) - V f(r; r_0, a_0)$$
$$- i(W - 4a_i W_s \frac{d}{dr_i}) f(r; r_i, a_i)$$
$$+ (\frac{\hbar}{m_\pi c})^2 V_{so} \frac{1}{r} \frac{d}{dr} f(r; r_{so}, a_{so}) \boldsymbol{\sigma} \cdot \boldsymbol{\ell},$$

$$f(r; r_0, a_0) = \frac{1}{1 + \exp(\frac{r - r_0 A_t^{1/3}}{a_0})}$$

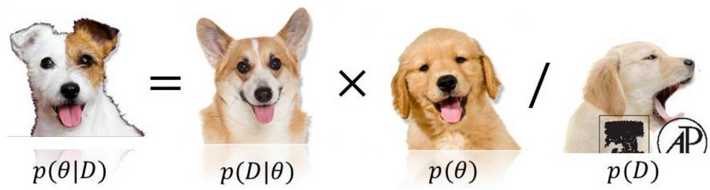
- > 6 free parameters per channel.
- Fit to elastic scattering data.

Elastic Data



- Same beam and target.
- Establish absolute scale.
- Constrain entrance parameters.

Bayesian Inference



- In our problem we try to learn about the parameters $\boldsymbol{\theta} = \{V_0, r_0, a_0, ...\}$
- Compare different ℓ values using evidence $P(D)$, which I'll call Z_ℓ
- Observed elastic scattering will give us the posterior (and, thus, uncertainty) $P(\boldsymbol{\theta}|D_{elastic})$

[1] *Doing Bayesian Data Analysis*, John K. Kruschke

The Method

PHYSICAL REVIEW C **102**, 024609 (2020)

Editors' Suggestion

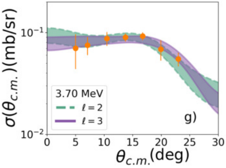
Bayesian analysis of the ^{70}Zn(d, ^{3}He) ^{69}Cu transfer reaction

C. Marshall,[1,2] P. Morfouace,[3] N. de Séréville,[4] and R. Longland[1,2]

[1]*Department of Physics, North Carolina State University, Raleigh, North Carolina 27695, USA*
[2]*Triangle Universities Nuclear Laboratory, Durham, North Carolina 27708, USA*
[3]*CEA, DAM, DIF, F-91297 Arpajon, France*
[4]*Université Paris-Saclay, Centre National de la Recherche Scientifique, IN2P3, IJCLab, 91405 Orsay, France*

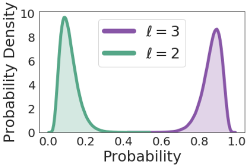
Parameter Estimation

$$P(C^2S|D_{Elastic}, D_{Transfer})$$



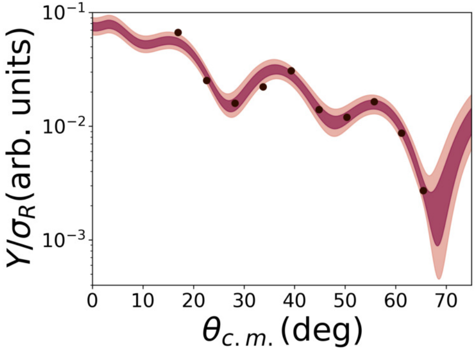
Model Comparison

$$P(\ell = 2) = Z_2/\sum_i Z_i$$



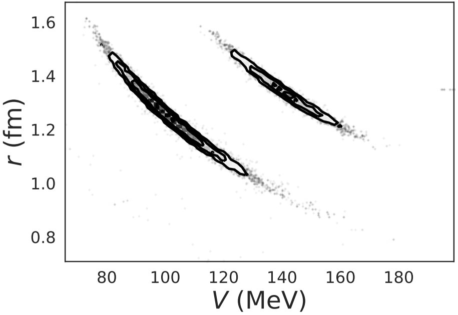
Elastic Results

- Posterior estimated using Markov chain Monte Carlo.
- Normalization estimated during fit.



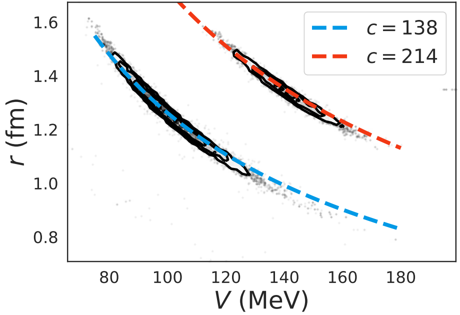
Discrete and Continuous Ambiguities

- Fairly dramatic for our ^{23}Na(^{3}He,^{3} He) data.
- $Vr^n = c$
- Higher angle data to remove?
- $\approx 7\%$ change in inferred normalization.



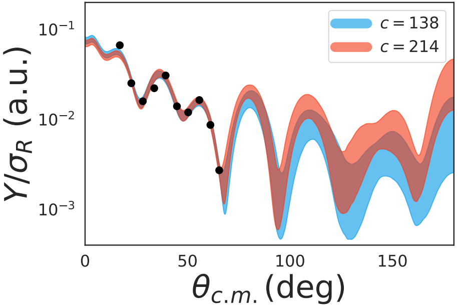
Discrete and Continuous Ambiguities

- Fairly dramatic for our ^{23}Na(^{3}He,^{3}He) data.
- $Vr^n = c$
- Higher angle data to remove?
- $\approx 7\%$ change in inferred normalization.



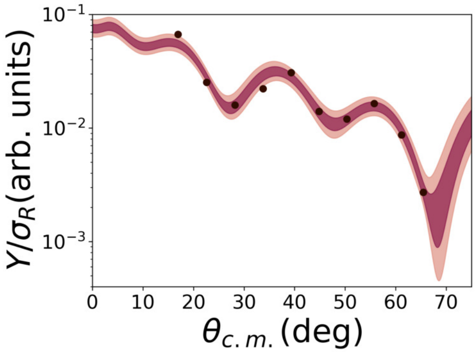
Discrete and Continuous Ambiguities

- Fairly dramatic for our ^{23}Na(^{3}He,^{3} He) data.
- $Vr^n = c$
- Higher angle data to remove?
- $\approx 7\%$ change in inferred normalization.

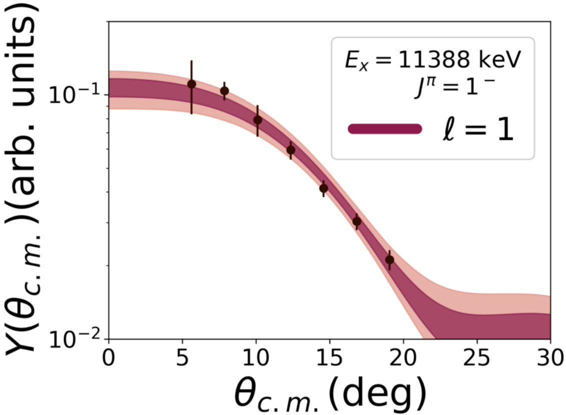


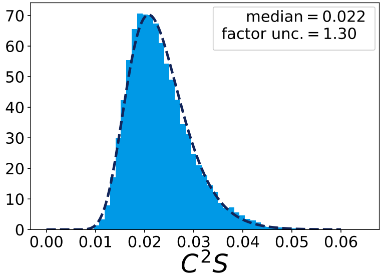
Making a Choice

- In a Bayesian framework we cannot just ignore this.
- Strictly enforce the mode that is consistent with the global data set.
- Uniform distribution $\pm 30\%$ of c.



Results


$E_x = 11388$ keV
$J^\pi = 1^-$
$\ell = 1$
$Y(\theta_{c.m.})$(arb. units)
$\theta_{c.m.}$(deg)
10^{-1}
10^{-2}
0
5
10
15
20
25
30

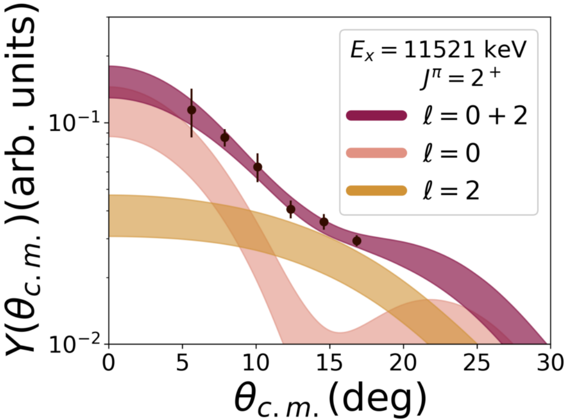

median = 0.022
factor unc. = 1.30
C^2S
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0
10
20
30
40
50
60
70

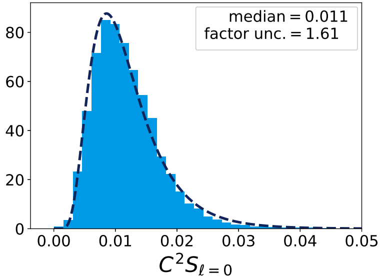
Multiple ℓ Values

$$\frac{d\sigma}{d\Omega}_{Exp} = C^2 S_{0+2}\left[\alpha \frac{d\sigma}{d\Omega}_{\ell=0} + (1-\alpha)\frac{d\sigma}{d\Omega}_{\ell=2}\right]$$

- ^{23}Na ground state is $J^\pi = 3/2^+$.
- Mixing seems to occur most often in $J^\pi = 2^+$ states ($\ell = 0 + 2$).
- Does this mixture imply greater uncertainty for C^2S?

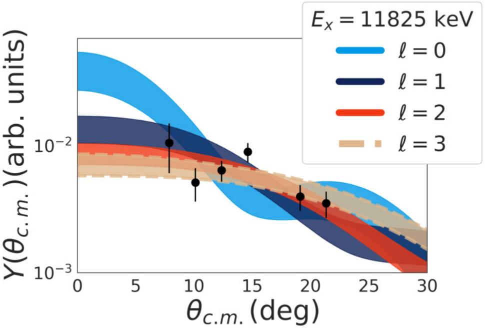
Additional Source of Uncertainty





132 keV Resonance

- 11825 keV state has unknown spin and parity.
- Direct measurement upper limits + indirect proton widths appear to rule out $\ell = 0, 1$ [1].
- LUNA recently measured $\omega\gamma$ at a significance of $< 3\sigma$ [2].
- Our results focuses on using just our values with probabilities for each possible ℓ value.

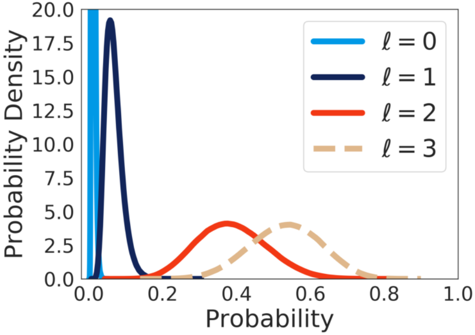


[1] J. M. Cesaratto et al., Phys. Rev. C, 88, 065806

[2] A. Boeltzig et al., Physics Letters B, 795, 122-128

132 keV Resonance

- 11825 keV state has unknown spin and parity.
- Direct measurement upper limits + indirect proton widths appear to rule out $\ell = 0, 1$ [1].
- LUNA recently measured $\omega\gamma$ at a significance of $< 3\sigma$ [2].
- Our results focuses on using just our values with probabilities for each possible ℓ value.

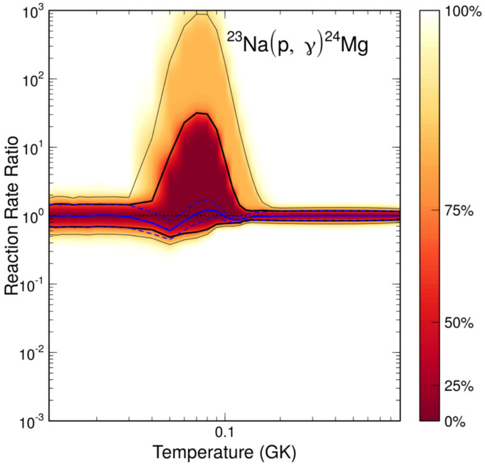


---

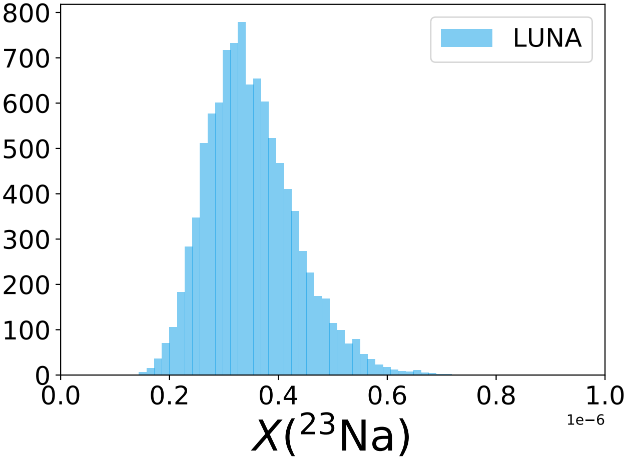
[1] J. M. Cesaratto et al., Phys. Rev. C, 88, 065806

[2] A. Boeltzig et al., Physics Letters B, 795, 122-128

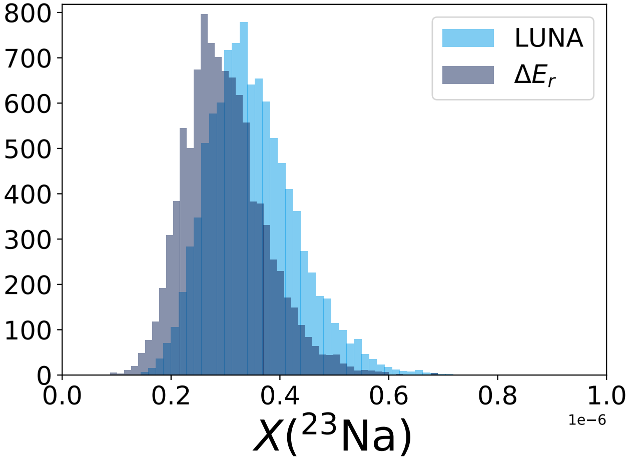
Astrophysical Impact



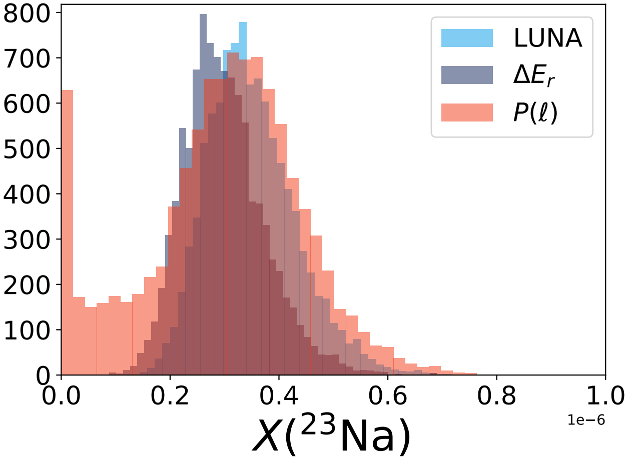
Astrophysical Impact



Astrophysical Impact



Astrophysical Impact



Summary

- Chemical signatures are a defining feature of globular clusters.

Summary

- Chemical signatures are a defining feature of globular clusters.
- Transfer reactions are essential tools for studying low-lying resonances.

Summary

- Chemical signatures are a defining feature of globular clusters.
- Transfer reactions are essential tools for studying low-lying resonances.
- Destruction of Na via ^{23}Na(p, γ) was previously underestimated due to error in energy calibration.

Summary

- Chemical signatures are a defining feature of globular clusters.
- Transfer reactions are essential tools for studying low-lying resonances.
- Destruction of Na via ^{23}Na(p,γ) was previously underestimated due to error in energy calibration.
- Optical potentials impact our ability to assign ℓ values and extract C^2S.

Summary

- Chemical signatures are a defining feature of globular clusters.
- Transfer reactions are essential tools for studying low-lying resonances.
- Destruction of Na via ^{23}Na(p, γ) was previously underestimated due to error in energy calibration.
- Optical potentials impact our ability to assign ℓ values and extract C^2S.
- These uncertainties can lead to even larger variations in the rate.

Thank You!

NCSU Group

- Richard Longland
- Kiana Setoodehnia (XFEL)
- Federico Portillo
- John Kelly

UNC Group

- Arthur E Champagne
- Christian Iliadis
- Robert Janssens
- Rafael S. de Souza
- Thomas Clegg

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under Award Numbers DE-SC0017799 and under Contract No. DE-FG02-97ER41041.